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**LETTER TO THE EDITOR**

# On the relaxation of nuclear polarization near 2D electron gas

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**Abstract.** We propose a treatment of relaxation of nuclear polarization in electronically poor systems, such as heterostructures, based on the consideration of electrons as provisional spin holders but not as the spin bath. In this case the rate $T_1^{-1}$ of decay of polarized states of nuclei is determined by the spin–orbital electron–phonon interaction and the conductivity of the electron gas (in open systems). The developed theory can be applied to experiments on nuclear magnetic relaxation in the quantum Hall effect regime.

Studies of nuclear magnetic resonance (NMR) have shown that it is a powerful tool for investigation of the electronic structure of metals [1]. In particular, information about the density and structure of electronic states is contained in the nuclear polarization relaxation rate $T_1^{-1}$. An application of NMR methods to the studies of 2D electron gas in the quantum Hall effect regime promises to provide a novel independent source of knowledge about this. In recent experiments [2, 3] the relaxation of polarization of nuclei covered by the wavefunctions of electrons localized inside a single GaAs–AlGaAs heterostructure was directly observed in the temporal evolution of the hyperfine interaction ($V_{ne} = \gamma_h(a^3/l_z)s_n s_e \delta(x - x')$) induced shift of the electron paramagnetic resonance frequency. As extracted rates have oscillated in a magnetic field (in a way similar to the Shubnikov–de Haas effect) it would be natural to treat this relaxation as a result of nuclei interactions with 2D electrons inside a quantum well (where $l_z$ is the width of the well).

Following the experience with NMR in metals one can consider [4–7] the observed relaxation as Korringa's, i.e. as a reorientation of a single nuclear spin due to interaction with electrons as the spin bath. This approach is based on the assumption that only the spin transfer from the nuclear to the electronic subsystem is a bottle-neck in the sequence of processes of spin flow far away from the heterostructure region. Generally, an alternative case can be viewed when the decay of excess spin is stopped by the saturation of electron gas polarization, so that only spin transfer out from the electron gas is the bottle-neck of the macroscopic depolarization process. That can be expected in carrier-poor systems with long-living electron spin excitations and low carrier mobilities. The 2D electron gas in a single heterostructure seems to provide an example of this kind

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because it contains many (suggestively polarized) nuclei, namely $(l_z/n_e a^3) \simeq 10^5$, per electron.

In the present letter we consider the relaxation of the macroscopic polarization of nuclei located in a quantum well with 2D electrons in a high magnetic field suggesting that electrons play the role only of provisional spin holders but not of a spin reservoir. From the point of view of arrangement of time scales, that means that spin transfer from many nuclei to a few electrons happens faster than the depolarization of the electronic gas due to spin–orbital electron–phonon collisions, and polarized carriers escape to metallic electrodes (in open systems), $\tau_K^{-1} \ll \tau_s^{-1}, \tau_D^{-1}$. On the other hand, we guess that electrons relax in energy within the shortest time scale, and we also completely neglect the influence of electron–electron interactions on the spin excitation formation.

Under the chosen approximations, spin-up as well as spin-down electrons can be considered in quasi-equilibrium with each one's own chemical potential, $\mu_+$ and $\mu_-$. The latter can be found from the stationary limit of spin balance equations for nuclear subsystems

$$(N_n/2)\frac{ds}{dt} = \int d\varepsilon\, w\nu_+(\varepsilon)\nu_-(\varepsilon)\tfrac{1}{4}\{(1+s)^2 n_f(\varepsilon-\mu_+)[1-n_f(\varepsilon-\mu_-)] - (1-s)^2 n_f(\varepsilon-\mu_-)[1-n_f(\varepsilon-\mu_+)]\} \tag{1}$$

and from the requirement that in the homogeneous case the partial polarization of electrons cannot change their summed local density,

$$\int d\varepsilon\,[\nu_+(\varepsilon)n_f(\varepsilon-\mu_+) + \nu_-(\varepsilon)n_f(\varepsilon-\mu_-)] = 0. \tag{2}$$

Here $\nu_+$ and $\nu_-$ are spin-up and spin-down electron densities of states at the Fermi level, $N_n (\sim l_z/a^3)$ and $s\,(-1 < s < 1)$ are the effective 2D density and the degree of polarization of spin $\frac{1}{2}$ nuclei [8]. $w \sim (\gamma_h a^3/l_z)^2/\hbar$ is the cross section of a spin-flip event due to the hyperfine interaction. The latter determines, in principle, Korringa's relaxation rate and it falls out from the final expressions in the quasi-equilibrium regime.

At low temperatures $T < \{\nu_+/(d\nu_+/d\varepsilon);\ \nu_-/(d\nu_-/d\varepsilon)\}$ and equations (1) and (2) give the following values of shifts $\mu_+ - \mu$ and $\mu_- - \mu$ of spin-up and spin-down quasi-equilibrium chemical potentials relative to the local value of a common potential $\mu$,

$$\mu_+ - \mu = 2T\frac{\nu_-}{\nu_- + \nu_+}\ln\left(\frac{1+s}{1-s}\right) \qquad \mu_- - \mu = -2T\frac{\nu_+}{\nu_- + \nu_+}\ln\left(\frac{1+s}{1-s}\right) \tag{3}$$

so that the maximal spin-capacitance of the electron gas is restricted by the value of an excess spin density $S_e = 4Ts\nu_+\nu_-/(\nu_+ + \nu_-)$. Therefore in the absence of any spin dumping by the electron gas itself the depolarization of nuclei will stop after a short time $t_* \sim 4\tau_K[T\nu_+\nu_-/(\nu_+ + \nu_-)N_n]$.

A weak inhomogeneity of spin distribution or a slow decay of spin polarization of electrons due to additional decay channels will disturb a local quasi-equilibrium and permit nuclei to continue to dump their excess spin. First, we consider the *effect of electron mobility on the dynamics of alignment of nuclear spins* taking into acount the currents of spin-up and spin-down electrons,

$$j_+ = \hat{\sigma}^+\nabla(\mu_+/e) \qquad j_- = \hat{\sigma}^-\nabla(\mu_-/e)$$

determined by the spatial dependence of their chemical potentials and by their conductivities $\hat{\sigma}^\pm$. As spin exchange among electrons and nuclei is guessed to be fast, the difference $\mu_+ - \mu_-$ is determined by the local polarization of nuclei and is equal to

$$\mu_+ - \mu_- = 2T\ln((1+s)/(1-s)) \approx 4Ts(x) \qquad s \ll 1. \tag{4}$$

On the other hand, electrical neutrality requires that

$$\operatorname{div}(j_+ + j_-) = \sigma_{xx}^+\nabla^2(\mu_+ - \mu)/e + \sigma_{xx}^-\nabla^2(\mu_- - \mu)/e + (\sigma_{xx}^+ + \sigma_{xx}^-)\nabla^2\mu/e = 0.$$

Therefore, the divergence of a spin current $j_s = (j_+ - j_-)/2e$ gives the diffusive contribution to the spin relaxation,

$$\operatorname{div} j_s = \frac{\sigma_{xx}^+\sigma_{xx}^-}{\sigma_{xx}^+ + \sigma_{xx}^-}\frac{4T}{e^2}\nabla^2 s(x,t) = 0. \tag{5}$$

Analogously, this also takes place in the problem of ambipolar diffusion [9]; an effective diffusion coefficient $D$, which can be derived from (5), is determined by the minimal conductivity of electrons of different spin projections and in the case of a narrow percolation edge is exponentially suppressed, $\exp(-g_eH/T) < D < \exp(-g_eH/2T)$, at all filling factors ($g_eH$ denotes Zeeman splitting).

*The spin-flip assisted electron–phonon interaction* provides another channel for the relaxation of local excess spin. We calculate its contribution to the spin balance from the electron–phonon collision integral summed over all electron energies using local quasi-equilibrium electron distribution functions $n_\pm(\varepsilon) = n_f(\varepsilon - \mu_\pm)$ with $\mu_+ - \mu_-$ given by equation (4),

$$\begin{aligned} I_s = \int \mathrm{d}\varepsilon\, \frac{1}{2}\left(\nu_+(\varepsilon)\frac{\mathrm{d}n_+(\varepsilon)}{\mathrm{d}t} - \nu_-(\varepsilon)\frac{\mathrm{d}n_-(\varepsilon)}{\mathrm{d}t}\right) \\ = -\frac{\pi}{\hbar}\int\int \mathrm{d}\varepsilon\,\mathrm{d}\varepsilon'\,\nu_+(\varepsilon)\nu_-(\varepsilon')|M_{+-}(\varepsilon,\varepsilon')|^2\nu_{ph}(|\varepsilon-\varepsilon'|) \\ \times\{\theta(\varepsilon'-\varepsilon)[n_+(\varepsilon)(1-n_-(\varepsilon'))f(\varepsilon'-\varepsilon) \\ -(1-n_+(\varepsilon))n_-(\varepsilon')(f(\varepsilon'-\varepsilon)+1)] \\ +\theta(\varepsilon-\varepsilon')[n_+(\varepsilon)(1-n_-(\varepsilon'))(f(\varepsilon-\varepsilon')+1) \\ -(1-n_+(\varepsilon))n_-(\varepsilon')f(\varepsilon-\varepsilon')]\}. \end{aligned} \tag{6}$$

Here $f(\omega)$ and $\nu_{ph}(\omega) \sim \omega^2$ are the distribution function and density of states of the phonons; $\theta(x) = 0, x < 0$ and $\theta(x) = 1, x > 0$. $M_{+-}(\varepsilon, \varepsilon')$ is the amplitude of the spin–orbital electron–phonon interaction that will be obtained for a crystal with a broken inversion symmetry [10] (GaAs is a material of this kind). In the frame of the model of a Gaussian random potential with a short correlation length it can be written as

$$|M_{+-}(\varepsilon,\varepsilon')|^2 = \frac{a^3}{2m_n\kappa^2}B^2|\varepsilon-\varepsilon'|/L^3$$

where $B$ is an interaction constant, $\kappa$ is an angle averaged sound velocity and $a$ and $m_n$ are the lattice constant and the unit-cell mass. The integration in (6) can be carried out for a sufficiently broadened Landau level, $\nu_\pm(\varepsilon) \sim \exp[-(\varepsilon - \varepsilon_\pm)^2/2\Gamma^2]$ [11, 12], at low temperatures $T < \Gamma$. We represent the result of this integration in terms of conventional constants of electron–phonon collisions in a semiconductor, such as the time $\tau_{ph}$ of deformation potential scattering of free electrons with an energy about the Debye energy, the ratio $\kappa m_e a/\hbar$ of Debye frequency $\omega_D \sim \kappa/a$ to the valence band width and

the ratio $\alpha$ of spin–orbit to spin-independent deformation potential electron–phonon interaction constants. For sufficiently weak polarizations, $s \ll 1$, we can find that the uniform contribution to the relaxation of local spin has the form

$$I_s = 71.4\, s\tau_{D,ph}^{-1}\, \alpha^2 \frac{(\kappa m_e a/\hbar)^{1/2}}{a^2} [\nu_+(\mu)\nu_-(\mu)\hbar^4/m_e^2]\left(\frac{T}{\hbar\omega_D}\right)^5. \tag{7}$$

Thus we see how electron diffusion driven spin current and phonon emission assisted decay of spin excitations in 2D electron gas contribute to the relaxation of spin polarization and together determine its evolution. The local spin balance, both of electrons and nuclei, leads to *the diffusion equation on nuclear polarization dynamics*

$$\partial_t s(x,t) = (2/N_n)\{\mathrm{div}\, j_s - I_s\} = D\nabla^2 s(x,t) - s\tau_s^{-1} \tag{8}$$

with the diffusion coefficient $D = 4(T/N_n e^2)[\sigma_{xx}^+\sigma_{xx}^-/(\sigma_{xx}^+ + \sigma_{xx}^-)]$ and uniform decay rate

$$\tau_s^{-1} = 35.7\tau_{ph}^{-1}\, \alpha^2[(\kappa m_e a/\hbar)^{1/2}/N_n a^2]\, [\nu_+(\mu)\nu_-(\mu)\hbar^4/m_e^2(T/\hbar\omega_D)^5.$$

In a sample joined with metallic electrodes this equation has to be completed by adding zero boundary conditions, so that the long-tail relaxation rate of polarization of nuclei in a finite open system (of characteristic dimensions $L$) can be written as

$$T_1^{-1} = \frac{4\pi}{L^2 N_n} \frac{\sigma_{xx}^+\sigma_{xx}^-}{(\sigma_{xx}^+ + \sigma_{xx}^-)(e^2/h)} \frac{T}{\hbar}$$
$$+ 35.7\, \tau_{ph}^{-1}\alpha^2 \frac{(\kappa m_e a/\hbar)^{1/2}}{N_n a^2} [\nu_+(\mu)\nu_-(\mu)\hbar^4/m_e^2]\left(\frac{T}{\hbar\omega_D}\right)^5. \tag{9}$$

It follows from (9) that both considered processes possess strong temperature dependences that can be distinguished experimentally from each other and from Korringa's relaxation, which shows a linear temperature dependence [3, 5, 7],

$$\tau_K^{-1} = \pi[\nu_+(\mu)\nu_-(\mu)\hbar^4/m_e^2](\gamma m_e/\hbar^2)^2 \frac{T}{\hbar}. \tag{10}$$

Moreover, the comparison between equations (9) and (10) indicates that at the lowest temperatures the spin transfer from electrons to the phonon bath happens slower than the pumping of electron spins by polarized nuclei, so that just equation (9) gives an observable relaxation time of a polarized state.

Finally, a rough estimation shows that in an infinite GaAs–AlGaAs heterostructure (with the well width $l_z \sim 10^{-6}$ cm) electrons work only as provisional spin holders and the spin-flip electron–phonon interaction really is a bottle-neck process which determines the observable nuclear spin relaxation time $t_1$ within the temperature interval $T < 10^{-2}\hbar\omega_D$ (both in weak and in strong magnetic fields), which corresponds to the $T < 1$ K regime in experimental systems [2, 3].

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